The prudential regulation framework of insurance businesses across the European Union (EU) will be radically transformed in the near future. Solvency II will replace the present framework, Solvency I, and is expected to come into force around 2010. Solvency II presumably represents a big step forward since it is a more transparent and risk oriented framework than Solvency I.

In the development of Solvency II there is a lot of discussion about the method used to calculate the risk margin of the insurance liabilities. The main purpose of the risk margin is to have the possibility to transfer the liabilities to a third party. It is a reflection of the non-hedgeable risks which the insurer is bearing. Several possible approaches are available to calculate the risk margin, but two methods are most common nowadays. These are the cost of capital and the percentile method. Both methods are recommended for the future calculation of the risk margin in the EU, the first by the Comité Européen des Assurances (CEA) and Chief Risk Officer (CRO) Forum and the second by the European Commission. There are many papers written about the pros and cons of both methods, however the relationship is still underexposed.

The main objective of this article is to obtain a mathematical relationship between the cost of capital and percentile approach, with the final goal of being able to quantify the difference between the two approaches for life as well as non-life insurers.

**TWO APPROACHES**

One of the possible approaches to determine the risk margin is the percentile approach. This method takes the perspective that the insurer must be able to meet its liability with some probability. The method focuses on the volatility of the distribution of the liabilities and is currently used in Australia. After identifying the distribution of the liabilities, the risk margin is computed by subtracting the best estimate from a predefined critical percentile value. In Australia this critical percentile is set to 75%. This leads to the following general formula for the risk margin under the percentile approach:

\[
\text{Risk Margin Percentile} = \frac{z_{\text{critical}} \cdot \delta_\text{total}}{1 + i_{\text{duration}}^{\text{duration}}} \tag{1}
\]

The second approach to calculate the risk margin is the cost of capital approach. The cost of capital approach takes the perspective that sufficient capital is needed to be able to run-off the business. The method focuses on the costs involved with providing solvency capital (SCR) to support the business-in-force until run-off. This is exactly the cost that is needed to transfer the liability to another party. The amount of solvency capital that a firm holds is invested in risk free investments and consequently yields a low profit. Because the investors demand a certain return higher than the risk free rate on all capital, the company is making a cost by holding the extra amount of capital. This is called the cost of capital.

There are different methods to calculate the risk margin based on the cost of capital approach. One of them is the Swiss Solvency Test (SST) which was set to be the benchmark cost of capital approach. The formula for the risk margin under the cost of capital method is:

\[
\text{Risk Margin Cost of Capital} = \text{CoC} \sum_{t=1}^{\text{run-off-period}} \frac{\text{SCR}(t)}{(1 + i)^t} \tag{2}
\]

For both approaches the purpose of the risk margin is exactly the same, that is to say having the possibility to transfer the liabilities to another party. When the purpose of both calculations is the same it is obvious that the results should also be approximately the same. When this assumption is made, it is possible to express the critical percentile value in a cost of capital percentage and the other way around which leads to:

\[
\frac{\text{CoC} \sum_{t=1}^{\text{run-off-period}} \frac{\text{SCR}(t)}{\text{BE}(t)(1 + i_{\text{duration}}^{\text{duration}})}}{\text{BE}(O)} \quad \frac{\delta_\text{total}}{(1 + i)^t} \frac{z_{\text{critical}}}{(1 + i_{\text{duration}}^{\text{duration}})} \tag{3}
\]

**ASSUMPTIONS**

The insurance companies in Europe all have different client files, what makes it difficult to make generalisations. The estimation of future cash flows is directly linked to the present client file and is necessary to identify, because the risk margin is based on this information. It is however not unrealistic to approximate the distribution of future cash flows by an exponential distribution. An exponential distribution is characterized by a high peak at the beginning of the probability distribution. The peak is followed by a fast decrease, but the probability
density function extinguishes very slowly. In the full paper a gamma distribution of future cash flows is also examined. Besides the distribution of the future cash flows it is also essential to make assumptions on the distribution of the liability. The expected cost can easily turn out to be totally different when the calibration of parameters is incorrect, a wrong functional form is used or a non expected shock occurs. The normal and the gamma distribution are examined as possible liability distributions.

RESULTS

By use of the assumptions as stated it is possible to calculate the risk margin under the cost of capital approach. The risk margin can be translated into a percentile value, leading to the same risk margin by using formula [3]. In this paper the Swiss cost of capital factor of 6% is used. Results are calculated for different durations and the different distributions for the liability. This leads for a life insurer with an exponential distribution of future cash flows to the following graph:

**Figure 1: Exponentially Distributed Future Cash Flows for a Life Insurer with a Normal and Gamma Distribution for the Liabilities**

![Graph showing the convergence of percentile values](image)

This graph shows a converging percentile value when the duration of the cash flows increases. The percentile value is more or less 75% if the duration is three and this percentile slowly decreases to 60%. It is important to note that the position of the normal curve as well as the gamma curve highly depend on the ratio of the volatilities of the different risk categories.

The volatility increases due to a higher run-off period, which is the main reason for the decrease of the percentile curve for the CoC method. Nonetheless, the decrease in percentile value becomes smaller and smaller. This is primarily due to the diminishing increase, or negative second derivative, of the run-off period. Another important reason for the shape of the percentile curve of the CoC method is the distribution of the liability. The difference of the two risk margins increases, but this increase is barely translated into a smaller percentile due to very small probabilities in the tail.

Besides the form, the graph also shows comparable results for the different liability distributions. Except for the very large durations the gamma distribution gives similar results to the normal distribution. The difference is very small because the volatility in proportion to the mean is small but rising. In a situation with a low duration the gamma distribution is hardly skewed thus almost the same as the normal distribution. When we enlarge the volatility, the difference for long durations between the gamma and normal distributions becomes significantly positive. The difference for long durations is positive since the gamma distribution is skewed to the right and the cost of capital risk margin consequently leads to a higher percentile.

For a non-life insurer with an exponential distribution of future cash flows the following graph results:

**Figure 2: Exponentially Distributed Future Cash Flows for a Non-Life Insurer with a Normal and Gamma Distribution for the Liabilities**

![Graph showing the convergence of percentile values](image)

For a non-life insurer with an exponential distribution of future cash flows the following graph results:
The graph shows a similar result as the results already acquired for the life insurer; the appropriate percentile decreases when the duration increases. However, the percentile curve for the non-life insurer is somewhat less steep. The percentile curve of the CoC method crosses the 75th percentile at a larger duration, because of the less negative derivative with respect to the duration. The percentile method leads under these circumstances to a lower risk margin than the cost of capital method, when the duration is smaller than about 5 years. A larger volatility leads to a larger difference between the gamma and exponential percentile curve, a similar result as for the life insurer.

Another important variable for a non-life insurer is the size of the total liability. The size of the liability is important because there is a non-linear relationship between the premium risk volatility and its solvency requirement. The unit of measurement becomes relevant in this situation, which is a very unwelcome consequence of the non-linear relationship. In this respect CEIOPS might want to reconsider the non-linear relationship and change this relation into a linear relation between the premium volatility and the appropriate solvency capital. Under the non-linear relationship the percentile curve of the CoC method shifts upward when the liability increases. Large non-life insurance companies with large amounts of future cash flows do therefore hold on to too much capital as a risk margin under the cost of capital method.

A CORRECTION TERM

The duration of the cash flows and in case of a non-life insurer the size of the liability, are the variables with a major influence on the appropriate percentile value corresponding to the cost of capital risk margin. The difference in every situation can easily be derived by subtracting the cost of capital risk margin from the risk margin stemming from the percentile approach. By carrying out a multivariate regression it is possible to define a formula which approximates the difference between the calculated risk margins.

For a life insurer the final equation for the correction term based on figure 3 is:

\[
Correction_{life,exp} = (1.6543 \times \text{duration} - 2.1876) \times \text{size}
\]

where size is the ratio of the liability of the insurer which correction term needs to be identified and the liability based on a fixed sum of cash flows used in this paper.

For a non-life insurer the following correction formula results:

\[
Correction_{non-life,exp} = (6.2379 \times \text{size} - 0.6533) \times \text{duration} - 60.174 \times \text{size} + 8.1589
\]

The correction term leads to much more comparable results and as a consequence is a good quantification of the difference between the risk margin calculated under the percentile and cost of capital approach.

It is important to note that the model used in this paper is a simplification of reality. The assumptions made in this article are certainly not applicable to every insurer, even though the assumptions are a best estimate of the insurance market. Nonetheless, this article gives a useful insight in the mathematical relationship between the cost of capital and percentile approach and moreover presents the quantification of the difference between the two approaches for the average life or non-life insurer.